TRANSVERSE FLOW OVER AN OSCILLATING PLATE WITH INJECTION OR SUCTION

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Transverse flow of an incompressible fluid over a plate oscillating in its own plane is analyzed for the case in which matter is injected or withdrawn through the surface. The problem is reduced to a system of ordinary differential equations, which is solved by the Newton method. It is shown that the flow can be represented as a superposition of steady and nonsteady fields. The propagation of velocity perturbations has the form of a damped wave at any frequencies. An approximative method is proposed for determining the frictional stress.

The analogous problem for an impervious plate has been treated in [1, 2].

In the present study we generalize the results of [1, 2] to the case of uniform suction from or injection into the boundary layer of a medium identical to the fluid flowing over the plate.

The velocity field is described by the system of equations and boundary conditions

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = U\frac{dU}{dx} + \nu\frac{\partial^2 u}{\partial y^2}, \quad \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$
$$u|_{y=0} = a\cos\omega t, \quad v|_{y=0} = v_W, \quad u|_{y\to\infty} = U = cx \tag{1}$$

The velocity components are sought in the form

$$u = cxf'(\eta) + a\sqrt{X^2+Y^2}\cos(\omega t + \varphi), \quad v = -\sqrt{c\nu}\, f(\eta)$$
$$\eta = \sqrt{\frac{c}{\nu}}\, y, \quad \cos\varphi = \frac{X}{\sqrt{X^2+Y^2}}, \quad \sin\varphi = \frac{Y}{\sqrt{X^2+Y^2}} \tag{2}$$

Assuming that the solution for the steady-state problem must result from (2) when $a = 0$, we have for the determination of $f(\eta)$

$$f''' + ff'' - f'^2 + 1 = 0$$

subject to the boundary conditions

$$f(0) = -v_W / \sqrt{c\nu} = -f_W, \quad f'(0) = 0, \quad f|'_{\eta\to\infty} = 1$$

The quantities X and Y are found from the system

$$X'' + f X' - f'X = -\omega c^{-1} Y, \quad Y'' + fY' - f'Y = \omega c^{-1} X$$
$$X(0) = 1, \quad X|_{\eta\to\infty} = 0, \quad Y(0) = 0, \quad Y|_{\eta\to\infty} = 0$$

The values of the lag angle φ are given in Table 1. The function $\varphi(\eta)$ is almost linear. It follows from the latter statement and the behavior of the amplitudes $\sqrt{X^2+Y^2}$ (Table 2) that the propagation of the velocity perturbations at any frequencies has the form of a damped wave analogous to a shear wave on a plate oscillating in a fluid at rest.

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TABLE 1

	$f_W = 1$			$f_W = -1$			
η	ω/c			η	ω/c		
	0.25	1.0	5.0		0.25	1.0	5.0
0.0	0.000	0.000	0.000	0.0	0.000	0.000	0.000
0.4	0.049	0.185	0.587	0.2	0.019	0.076	0.281
0.8	0.095	0.359	1.161	0.4	0.038	0.147	0.553
1.2	0.139	0.525	1.725	0.6	0.055	0.215	0.818
1.6	0.180	0.685	2.282	0.8	0.071	0.279	1.078
2.0	0.220	0.838	2.833	1.0	0.087	0.341	1.332
2.4	0.258	0.985	3.376	1.2	0.102	0.401	1.581
2.8	0.293	1.124	3.909	1.4	0.117	0.459	1.826
3.2	0.326	1.256	4.429	1.6	0.131	0.515	2.065
3.6	0.358	1.378	4.934	1.8	0.144	0.568	2.299
4.0	0.387	1.493	5.421	2.0	0.157	0.620	2.527

TABLE 2

	$f_W = 1$			$f_W = -1$			
η	ω/c			η	ω/c		
	0.25	1.0	5.0		0.25	1.0	5.0
0.0	1.0000	1.0000	1.0000	0.0	1.0000	1.0000	1.0000
0.4	0.8479	0.8198	0.6249	0.2	0.7244	0.7182	0.6436
0.8	0.6722	0.6316	0.3738	0.4	0.5087	0.5006	0.4059
1.2	0.4965	0.4548	0.2116	0.6	0.3464	0.3387	0.2503
1.6	0.3393	0.3039	0.1123	0.8	0.2287	0.2224	0.1507
2.0	0.2128	0.1869	0.0554	1.0	0.1464	0.1416	0.0885
2.4	0.1214	0.1048	0.0252	1.2	0.0908	0.0874	0.0506
2.8	0.0624	0.0531	0.0105	1.4	0.0545	0.0523	0.0282
3.2	0.0286	0.0241	0.0040	1.6	0.0317	0.0303	0.0153
3.6	0.0117	0.0097	0.0014	1.8	0.0178	0.0170	0.0081
4.0	0.0042	0.0035	0.0004	2.0	0.0097	0.0092	0.0041

The suppressant action of the external flow in the investigated process limits the velocity at a fixed point in space to the value

$$A(\eta, f_W) = \sqrt{X^2 + Y^2}\,|_{\omega/c=0} \tag{3}$$

If the frequency is so high that the velocity perturbations do not have time to reach values consistent with Eq. (3), the amplitudes are dictated by the same factors as a shear wave. The nonsteady component can be approximated by the relation [3]

$$u_t = a\exp\left[-n(\omega/c,\ f_W)\,\eta\right]\cos\left[\omega t - m(\omega/c,\ f_W)\ \ \eta\right] \tag{4}$$

$$n = m = \sqrt{\omega/2c} \quad \text{for} \quad f_W = 0 \tag{5}$$

It is justifiable to use Eq. (4) if $\exp(-n\eta) < A(\eta, f_W)$. When the opposite condition holds, relation (4) must be replaced by the relation

$$u_t = aA(\eta,\ f_W)\cos(\omega t - m\eta)$$

The dimensionless frictional stress at low frequencies is written in the form

$$\frac{1}{a}\frac{\partial u_t}{\partial \eta}\bigg|_{\eta=0} = A'(0, f_W)\cos\omega t + m\sin\omega t \tag{6}$$

and at high frequencies in the form

$$\frac{1}{a}\frac{\partial u_t}{\partial \eta}\bigg|_{\eta=0} = -n\cos\omega t + m\sin\omega t \tag{7}$$

A suitable candidate for the threshold frequency separating these intervals is the frequency ω_0 at which

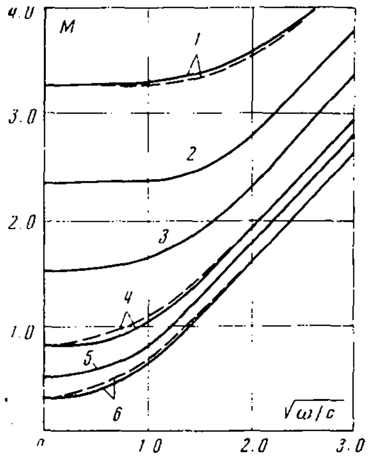

Fig. 1

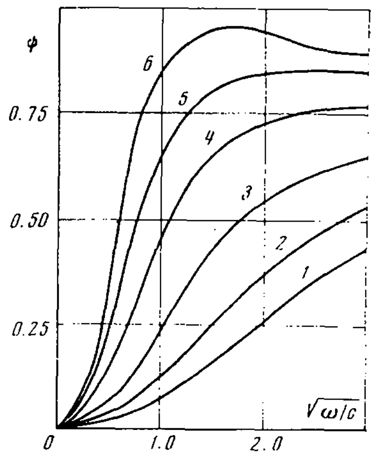

Fig. 2

the exponential in Eq. (4) coincides with the function $A(\eta, f_W)$ near the surface. The indicated value is given by the equation

$$\left\{\frac{\partial}{\partial\eta}\exp\left[-n(\omega_0, f_W)\,\eta\right]\right\}_{\eta=0} = -n(\omega_0, f_W) = A'(0, f_W) \tag{8}$$

The solid curves in Fig. 1 represent the frictional stress amplitude calculated by means of the exact solution according to the expression

$$\left.\frac{\partial u}{\partial\eta}\right|_{\eta=0} = cxf''(0) + aM\cos(\omega t + \psi + \pi), \quad M = \sqrt{X'^2(0) + Y'^2(0)}$$

and the dashed curves represent the same quantity calculated by the approximative method, i.e., according to Eq. (6) for $\omega<\omega_0$ and according to Eq. (7) for $\omega>\omega_0$. The curves are numbered as follows: 1) $f_W = -3$; 2) -2; 3) -1; 4) 0; 5) 0.5; 6) 1.

For $\omega = 0$ we have $m = 0$, the motion is quasi-steady, and the frictional stress varies in phase with the velocity of the surface. For $\omega \neq 0$ the oscillations of the peripheral fluid layers lag behind the oscillations of layers closer to the surface. This effect yields an additional frictional stress component $m \sin(\omega t)$.

At very low frequencies $m \ll A'(0, f_W)$, the growth of the frictional stress amplitude is slower, and then it accelerates. This growth is attributable to the increase in the lag angles with frequency.

For $\omega > \omega_0$ both components of the velocity gradient increase. Consequently, the growth of the indicated amplitudes becomes faster. At $f_W = 0$ the frictional stress amplitude is proportional to $\sqrt{\omega/c}$. For $f_W < 0$ the velocity amplitudes decrease, and the absolute value of the derivative $A'(0, f_W)$ increases. The frictional stress is therefore greater with suction than in the case $f_W = 0$. For higher frequencies ω_0 Eq. (8) is satisfied, so that the low-frequency zone extends higher. Injection has the opposite effect.

Inasmuch as suction increases the role of the first coefficient in Eq. (6), the lead angle ψ increases more slowly in this case (Fig. 2). With injection the lead angle increases so rapidly that for a certain value of $\sqrt{\omega/c}$ it exceeds the limiting value corresponding to $\sqrt{\omega/c} \to \infty$.

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